FILTRATION

Determination of the filtration constants

These constants are product of specific cake resistance and concentration, i.e. $\alpha \cdot c$, (if concentration c is constant) and media resistance R_m.

For determining these constants, the following straight line is fitted to the measurement data:

$$\frac{\Delta t}{\Delta V} = a \cdot V + b$$

The constants can be calculated from the slope a and intersetion b, according to the following formulas:

$$a = \frac{\alpha \cdot c \cdot \eta}{A^2 \cdot \Delta p} \qquad b = \frac{R_m \cdot \eta}{A \cdot \Delta p}$$

Filtration time

Time needed for filtering a given volme V:

$$t = \frac{\eta}{\Delta p} \cdot \left[\frac{\alpha \cdot c}{2} \cdot \left(\frac{V}{A} \right)^2 + R_m \cdot \frac{V}{A} \right]$$

Optimal filtrate volume and optimal filtration time

$$V_{opt} = A \cdot \sqrt{\frac{2 \cdot \Delta p \cdot t_{ch}}{\eta \cdot \alpha \cdot c}}$$

$$t_{opt} = t_{ch} + R_m \cdot \frac{\eta}{\Delta p} \cdot \sqrt{\frac{2 \cdot \Delta p \cdot t_{ch}}{\eta \cdot \alpha \cdot c}}$$

(t_{ch}: changeover time)

Explanations

$$\frac{dt}{dV} = a \cdot V^2 + b$$

$$t = \frac{a}{2} \cdot V^2 + b \cdot V$$

$$\frac{V}{t + t_{ch}} \to \max \;\Rightarrow\; \varphi = \frac{t + t_{ch}}{V} \to \min$$

$$\varphi = \frac{t + t_{ch}}{V} = \frac{a}{2} \cdot V + b + \frac{t_{ch}}{V}$$

$$\frac{d\varphi}{dV} = \frac{a}{2} - \frac{t_{ch}}{V^2} = 0 \;\Rightarrow\; V_{opt}^2 = \frac{2t_{ch}}{a}$$

$$t_{opt} = \frac{a}{2} \cdot V_{opt}^2 + b \cdot V_{opt} \;\Rightarrow\; t_{opt} = t_{ch} + \frac{b}{a} \cdot \sqrt{2t_{ch}}$$

This t_{opt} does **not** includes t_{ch}; **optimal cycle time is $t_{opt}+t_{ch}$.**

Problem 1

The following data have been measured during filtration of chalk with a plates and frames filter press of area 1600 cm^2 with pressure drop $7.848 \cdot 10^4$ Pa:

V [liter]	5	10	15	20	25	30
t [min]	0.8	1.8	3.05	4.55	6.25	8.15

Dynamic viscosity of the filtrate is 10^{-3} Pas.

Determine:

a) Filtration constants
b) Filtration time for filtering 500 liter over a filter press of area 1 m^2 with pressure drop $1.6 \cdot 10^5$ Pa
c) Assuming 6 min changeover time, the number of batches needed to filter 100 liter on the original filter press
d) The time of filtering 100 liter according to problem c)
e) The suspension to be filtered contains 90 kg chalk powder per cubic meter. The total amount is filtered out. Assume a filter press with a 3 cm wide frame. How much percent does the cake fill the frame if one frame and two ribbed part are used? Assume a cake with 1700 kg/m^3.

Soluton

$A = 1600\ \text{cm}^2 = 0.16\ \text{m}^2$
$\Delta p = 7.848 \cdot 10^4$ Pa
$\eta = 10^{-3}$ Pas

a) Filtration constants

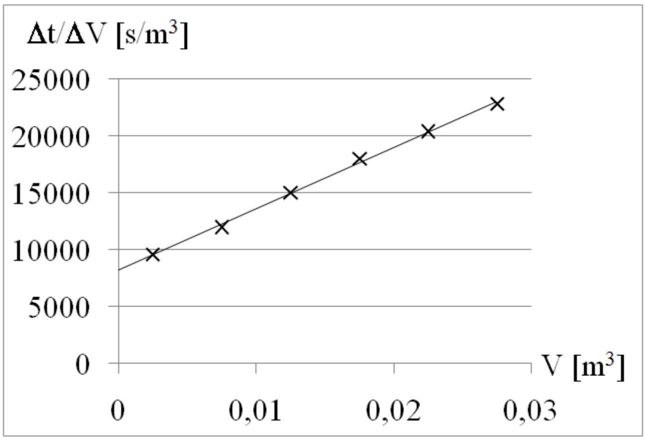
Compute the points for plotting $\Delta t/\Delta V$ against V:

V [liter]	5	10	15	20	25	30
t [min]	0.8	1.8	3.05	4.55	6.25	8.15
V [m^3]	0.0025	0.0075	0.0125	0.0175	0.0225	0.0275
$\Delta t/\Delta V$ [s/m^3]	9600	12000	15000	18000	20400	22800
$\overset{*}{V}_n$	2.5	7.5	12.5	17.5	22.5	27.5

The calculated values are plotted againts the centerpoints of the V-intervals.
That is, the co-ordinates of the nth point are:

$$\overset{*}{V}_n = \frac{V_{n-1} + V_n}{2} \quad \text{and} \quad \left(\frac{\Delta t}{\Delta V}\right)_n = \frac{t_n - t_{n-1}}{V_n - V_{n-1}}$$

When you have time, statistical regression is applied. For a fast estimate, however, fitting a straight line to two selected points may be applied. Avoid selecting the first point, if possible, because it usually accumulates large uncertainty.



Here a line is fitted to points 2 and 5.
The slope and one of the constants:

$$a = \frac{\Delta \frac{\Delta t}{\Delta V}}{\Delta V} = \frac{20400\frac{s}{m^3} - 12000\frac{s}{m^3}}{0.0225m^3 - 0.0075m^3} = 5.6\cdot 10^5\ \frac{s}{m^6}$$

$$a = \frac{\alpha \cdot c \cdot \eta}{A^2 \cdot \Delta p}$$

$$\alpha \cdot c = \frac{a \cdot A^2 \cdot \Delta p}{\eta} = \frac{5.6\cdot 10^5\ \frac{s}{m^6}\cdot\left(0{,}16m^2\right)^2\cdot 7.848\cdot 10^4\,Pa}{10^{-3}\,Pas} = 1.125\cdot 10^{12}\ \frac{1}{m^2}$$

Intersection with the axis is obtained by substituting the slope and data of point 2 to the equation of the straight line:

$$\frac{\Delta t}{\Delta V} = a \cdot V + b$$

$$b = \frac{\Delta t}{\Delta V} - a\cdot V = 12000\frac{s}{m^3} - 5.6\cdot 10^5\ \frac{s}{m^6}\cdot 0.0075m^3 = 7800\frac{s}{m^3}$$

$$b = \frac{R_m \cdot \eta}{A \cdot \Delta p}$$

$$R_m = \frac{b \cdot A \cdot \Delta p}{\eta} = \frac{7800\frac{s}{m^3}\cdot 0.16m^2\cdot 7.848\cdot 10^4\,Pa}{10^{-3}\,Pas} = 9.8\cdot 10^{10}\ \frac{1}{m}$$

b) Filtration time for filtering 500 liter over a filter press of area 1 m^2 with pressure drop $1.6\cdot 10^5$ Pa

A’ = 1 m^2
Δp’ = $1.6\cdot 10^5$ Pa
V’ = 500 l = 0.5 m^3
t’ = ?

$$t' = \frac{\eta}{\Delta p'}\cdot\left[\frac{\alpha\cdot c}{2}\cdot\left(\frac{V}{A'}\right)^2 + R_m\cdot\frac{V}{A'}\right]$$

$$t' = \frac{10^{-3}\,Pas}{1.6\cdot 10^5\,Pa}\cdot\left[\frac{1.125\cdot 10^{12}\ \frac{1}{m^2}}{2}\cdot\left(\frac{0.5m^3}{1m^2}\right)^2 + 9.8\cdot 10^{10}\ \frac{1}{m}\cdot\frac{0.5m^3}{1m^2}\right] = 1185\,s = 19.75\,min$$

c) Assuming 6 min changeover time, the number of batches needed to filter 100 liter on the original filter press

$V'' = 100$ liter
$t_{ch} = 6$ min $= 360$ s

Optimal filtrate volume is to be determined. The root member in the formula is worth to compute separately because it is used in the optimal filtration time, too.

$$\sqrt{\frac{2\cdot\Delta p\cdot t_{ch}}{\eta\cdot\alpha\cdot c}} = \sqrt{\frac{2\cdot 7.848\cdot 10^4\,\mathrm{Pa}\cdot 360\mathrm{s}}{10^{-3}\,\mathrm{Pas}\cdot 1.125\cdot 10^{12}\,\frac{1}{\mathrm{m}^2}}} = 0.224\mathrm{m}$$

$$V_{opt} = A\cdot\sqrt{\frac{2\cdot\Delta p\cdot t_{ch}}{\eta\cdot\alpha\cdot c}} = 0.16\mathrm{m}^2\cdot 0.224\mathrm{m} = 0.036\mathrm{m}^3 = 36\ \text{liter}$$

Number of batches:

$$n = \frac{V''}{V_{opt}} = \frac{100\ \text{liter}}{36\ \text{liter}} = 2.78$$

Round upwards: n=3.

d) The time of filtering 100 liter according to problem c)

Two whole batch and a shorter batch is enough (n=2.78). The device must be cleaned after the last, shorter, operation as well, i.e. 3 changeover periods must be taken into account.

$$t_{total} = \lfloor n\rfloor\cdot t_{opt} + t_{remained} + \lceil n\rceil\cdot t_{ch} = 2\cdot t_{opt} + t_{remained} + 3\cdot t_{ch}$$

Optimal filtration time:

$$t_{opt} = t_{ch} + R_m\cdot\frac{\eta}{\Delta p}\cdot\sqrt{\frac{2\cdot\Delta p\cdot t_{ch}}{\eta\cdot\alpha\cdot c}} = 360\mathrm{s} + 9.8\cdot 10^{10}\,\frac{1}{\mathrm{m}}\cdot\frac{10^{-3}\,\mathrm{Pas}}{7.848\cdot 10^4\,\mathrm{Pa}}\cdot 0.224\mathrm{m} = 640\mathrm{s}$$

Remaining volume for the third batch:

$$V_{remained} = V - 2\cdot V_{opt} = 100\,\text{liter} - 2\cdot 36\,\text{liter} = 28\,\text{liter} = 0.028\mathrm{m}^3$$

Filtration time of the third batch:

$$t_{remained} = \frac{\eta}{\Delta p}\cdot\left[\frac{\alpha\cdot c}{2}\cdot\left(\frac{V_{remained}}{A}\right)^2 + R_m\cdot\frac{V_{remained}}{A}\right]$$

$$t_{remained} = \frac{10^{-3}\,\mathrm{Pas}}{7.848\cdot 10^4\,\mathrm{Pa}}\cdot\left[\frac{1.125\cdot 10^{12}\,\frac{1}{\mathrm{m}^2}}{2}\cdot\left(\frac{0.028\mathrm{m}^3}{0.16\mathrm{m}^2}\right)^2 + 9.8\cdot 10^{10}\,\frac{1}{\mathrm{m}}\cdot\frac{0.028\mathrm{m}^3}{0.16\mathrm{m}^2}\right] = 438\mathrm{s}$$

Total time:

$$t_{total} = 2\cdot t_{opt} + t_{remained} + 3\cdot t_{ch} = 2\cdot 640\mathrm{s} + 438\mathrm{s} + 3\cdot 360\mathrm{s} = 2798\mathrm{s} = 46.64\,\mathrm{min}$$

e) The suspension to be filtered contains 90 kg chalk powder per cubic meter. The total amount is filtered out. Assume a plates and frames filter press with a 3 cm wide frame. How much percent does the cake fill the frame if one frame and two ribbed part are used? Assume a cake with 1700 kg/m^3.

c = 90 kg/m^3
h_{frame} = 2 cm
ρ_{cake} = 1700 kg/m^3

Mass of cake after one batch, i.e. V_{opt} = 0.036 m^3 filtrate:

$$m_{cake} = V_{opt} \cdot c = 0.036 m^3 \cdot 90 \frac{kg}{m^3} = 3.24 kg$$

Cake volume:

$$V_{cake} = \frac{m_{cake}}{\rho_{cake}} = \frac{3.24 kg}{1700 \frac{kg}{m^3}} = 1.9 \cdot 10^{-3} m^3$$

Height of the cake on the filtering area:

$$h_{cake} = \frac{V_{cake}}{A} = \frac{1.9 \cdot 10^{-3} m^3}{0.16 m^2} = 0.012 m$$

Cake is formed on both sides of the frame. Thus, the ratio of filling the place is

$$x = \frac{2 \cdot h_{cake}}{h_{frame}} = \frac{2 \cdot 0.012 m}{0.03 m} = 0.8$$

This is 80 %.